

# Cures for Mechanical Resonance in Industrial Servo Systems.

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## Abstract

Mechanical resonance is a pervasive problem in servo systems. Most problems of resonance are caused by the compliance of power transmission components. Standard servo control laws are structured for rigidly-coupled loads. However, in practical machines some compliance is always present; this compliance often reduces stability margins, forcing servo gains down and reducing machine performance.

Mechanical resonance falls into two categories: low-frequency and high-frequency. High-frequency resonance causes instability at the natural frequency of the mechanical system, typically between 500 and 1200 Hz. Low-frequency resonance occurs at the first phase crossover, typically 200 to 400 Hz. Low-frequency resonance occurs more often in general industrial machines. This distinction, rarely made in the literature, is crucial in determining the most effective means of correction. This paper will present several methods for dealing with low-frequency resonance, all of which are compared with laboratory data.

## Introduction

It is well known that servo performance is enhanced when control-law gains are high. However, instability results when a high-gain control law is applied to a compliantly-coupled motor and load. Machine designers specify transmission components, such as couplings and gearboxes, to be rigid in an effort to minimize mechanical compliance. However, some compliance is unavoidable. In addition, marketplace limitations, such as cost and weight, force designers to choose lighter-weight components than would otherwise be desirable. Often, the resulting rigidity of the transmission is so low that instability results when servo gains are raised to levels necessary to achieve desired performance.

The well-known lumped-parameter model for a compliantly-coupled motor and load is shown in Figure 1 and a block-diagram is shown in Figure 2[1]. The load and motor are two independent inertias connected by non-rigid components. The equivalent spring constant of the entire transmission is  $K_S$ ; also a viscous damping term,  $b_S$ , is shown in Figure 2, which produces torque in proportion to the velocity difference of motor and load.

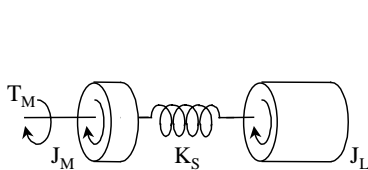


Figure 1. Simple compliantly-coupled motor and load

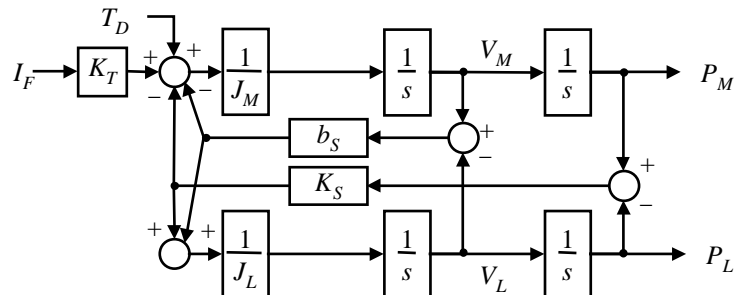


Figure 2. Block diagram of compliantly-coupled load

## Two-part transfer function

The transfer function from drive current,  $I_F$ , to motor velocity,  $V_M$ , is

$$\frac{V_M(s)}{I_F(s)} = \left( \frac{K_T}{J_M + J_L} \frac{1}{s} \right) \left( \frac{J_L s^2 + b_S s + K_S}{(J_L J_M / (J_L + J_M)) s^2 + b_S s + K_S} \right) \quad (1)$$

(1) has two terms. The term on the left is a rigidly-coupled motor and load and the term on the right is the effect of the compliant coupling. Note that (1) represents the plant in the case where the feedback sensor is on the motor (as opposed to the load), as is common in industry.

The ideal plant for traditional control laws is a scaled integrator. The term on the left of (1) is such a plant. However, that ideal plant is corrupted by the compliance term (right side of (1)). Figure 3 shows a Bode plot of (1) where the load has about five times more inertia than the motor. The compliance term has a gain peak at the

resonant frequency,  $F_R$ , and a gain minimum at the anti-resonant frequency,  $F_{AR}$ , as shown in (2). The corrupting effect of compliance can be seen in Figure 3. Were the load rigidly-coupled, the plant would be the ideal integrator (the left side of (1)) which is shown as a dashed line. However, compliance causes attenuation at and around  $F_{AR}$ , and amplification at, around, and above  $F_R$ .

$$F_R = \frac{1}{2\pi} \sqrt{\frac{K_S}{J_P}} \text{ Hz}, F_{AR} = \frac{1}{2\pi} \sqrt{\frac{K_S}{J_L}} \text{ Hz}. \quad (2)$$

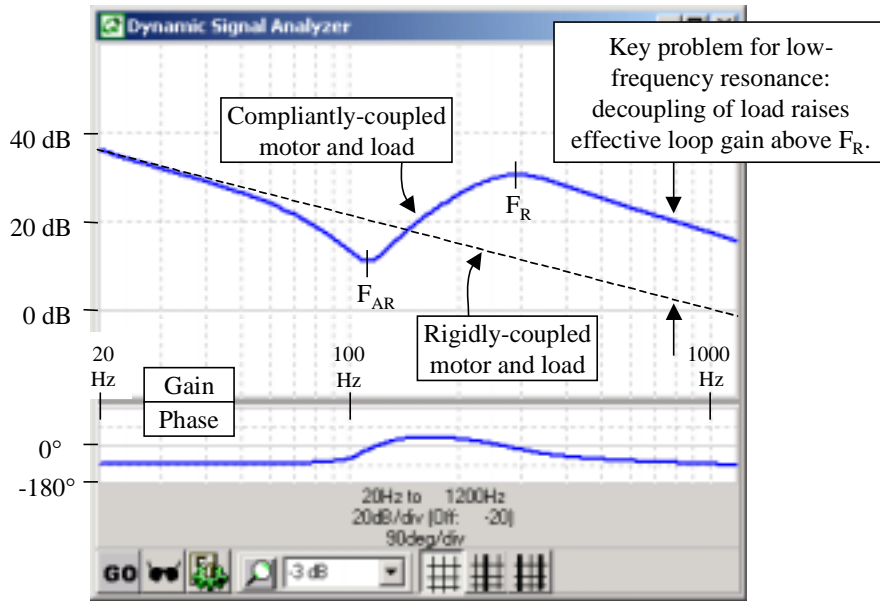


Figure 3. Plot of motor/load plant with 5:1 load-to-motor inertia ratio.

### Low-frequency resonance

The key problem in low-frequency resonance is the increase in gain at frequencies above  $F_R$  [1]. As shown in Figure 3, below  $F_{AR}$ , the transfer function acts like a simple integrator. The gain falls at 20dB/decade and the phase is approximately  $-90^\circ$ . It also behaves like an integrator above  $F_R$ , but with a gain substantially increased compared to the gain well below  $F_{AR}$ . Above  $F_R$ , the load is effectively disconnected from the motor so that the gain of the plant is the inertia of the motor. In Figure 3, which has a 5:1 load-to-inertia ratio, the effective inertia falls by a factor of 6 or 6 dB. This raises the loop gain by 16 dB at high frequencies, reducing margins of stability.

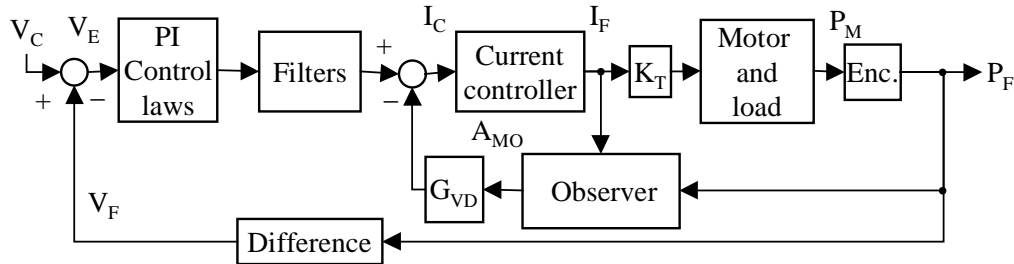


Figure 4. Velocity control system.

### Velocity control law

Figure 4 shows a velocity control system. The velocity error ( $V_E$ ) is processed by a control law and filters. The current command,  $I_C$ , is connected to the current controller, which produces current,  $I_F$ , in the motor. The motor/load plant is connected to an encoder. An observer, fed by the feedback current and position, produces an

observed acceleration,  $A_{MO}$ . The observer will be discussed in detail later. With exception to the observer, Figure 4 represents a velocity control loop as is commonly used in industry.

The problem caused by compliant (right) term in (1) is seen in the open-loop Bode plot of the velocity controller of Figure 4. The open-loop transfer function,  $V_F(s)/V_E(s)$ , is well-known to predict stability problems using two measures: phase margin (PM) and gain margin (GM). PM is the difference of  $-180^\circ$  and the phase of the open-loop at the frequency where the gain is 0 dB. GM is the negative of the gain of the open loop at the frequency where the phase crosses through  $-180^\circ$ .

The open loop plot for a rigidly-coupled load demonstrating low-frequency resonance is shown in Figure 5. The harmful effects of compliance are seen in the GM. As marked in Figure 5, when the resonant frequency is well below the first phase crossover (270 Hz) the effect of the compliant load is to reduce the GM approximately by the amount  $(J_M + J_L)/J_M$ . If  $J_L/J_M$  (the “inertia mismatch”) is 5, the reduction of GM will be 6 or about 16 dB. Assuming no other remedy were available, the gain of the compliantly-coupled system would have to be reduced by 16 dB, compared to the rigid system, assuming both would maintain the same GM. Such a large reduction in gain would translate to a system with much poorer command and disturbance response.

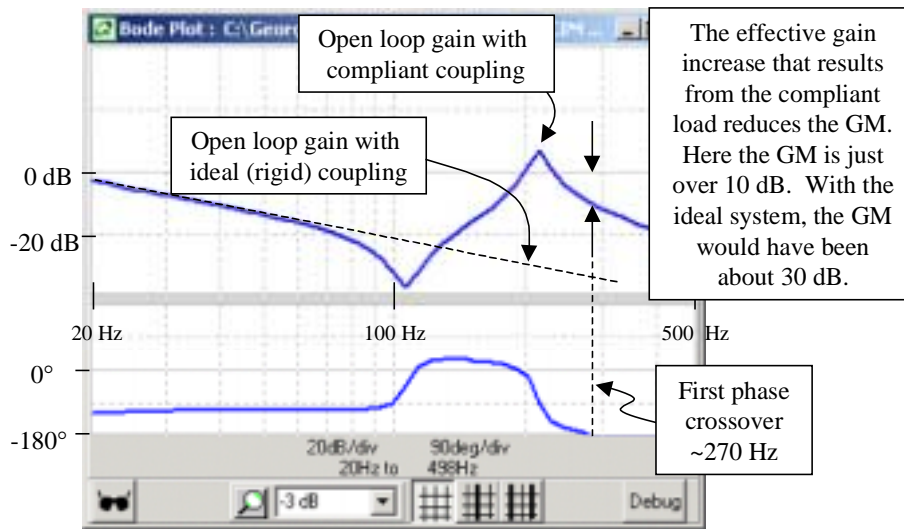


Figure 5. Open loop with low-frequency resonance.

High-frequency resonance is different; it occurs in lightly damped mechanisms when the natural frequency of the mechanical system ( $F_R$ ), is well above the first phase crossover. Here, the gain near  $F_R$  forms a strong peak. The gain caused by a lightly damped right-hand term in (1) can exceed 40dB. While both types of resonance are caused by compliance, the relationship of the  $F_R$  and the first phase crossover changes the remedy substantially; cures of high-frequency resonance can exacerbate problems with low-frequency resonance. The mechanical structures that cause high-frequency resonance (stiff transmission components and low damping) are typical of high-end servo machine such as machine tools. Smaller and more cost sensitive general-purpose machines in such industries as packaging, textiles, plotting, and medical, typically have less rigid transmissions and higher damping so that low-frequency resonance is more common.

### Methods of correction applied to low-frequency resonance.

Numerous methods have been used to remedy of resonance [1 - 5]. The most common in industry is the low-pass filter. Another method suggested by numerous authors is acceleration feedback, where the acceleration is provided by an observer. This paper will not discuss the notch filter, which is effective with high-frequency resonance, but not with low-frequency resonance, a problem that appears over a broad frequency range.

### Test Unit

The test unit, shown in Figure 6, is a motor and load connected by plastic tubing that has been slit to increase compliance. The motor inertia is  $1.8 \times 10^{-5} \text{ kg-m}^2$  and the load is  $6.3 \times 10^{-5} \text{ kg-m}^2$  (both include the coupling to the tubing). The coupling has a compliance of 30 Nm/rad. This ratio produced a resonant frequency of about 230 Hz and an anti-resonant frequency of about 110 Hz. These figures are consistent with a machines used in industry. The drive used was a 3 Amp Kollmorgen ServoStar 600 amplifier executing the velocity loop

at 16 kHz. This drive is equipped with low-pass and bi-quad filters, an observer, and acceleration feedback.

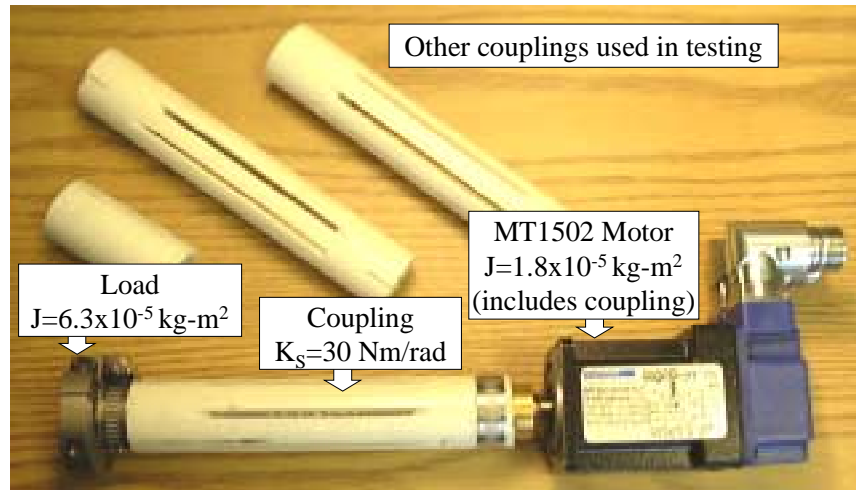


Figure 6. Test-unit mechanism is a scaled model of machines commonly used in industry.

### Baseline System

The baseline system had no filters and did not use acceleration feedback. The PI controller was tuned to maximize performance. The proportional gain was raised as high as possible without generating large oscillations ( $G_V = 2.5$ ). The integral gain was then raised until a step command generated 25% overshoot ( $G_{VTN} = 10$  mSec). The step response is shown in Figure 7. Settling time was about 60 mSec. The -3dB bandwidth was measured as 23 Hz.

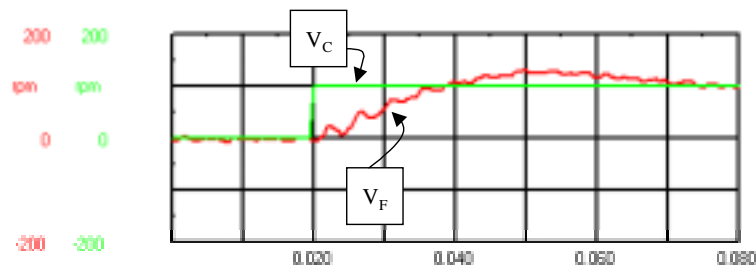


Figure 7. Step response of baseline system.

### Low-Pass Filter

A single-pole low-pass filter was used. The frequency was adjusted to 50 Hz. Servo gains could be raised to  $G_V = 5$  and  $G_{VTN} = 17$  mSec. The settling time improved to 35 mSec. The bandwidth was 35 Hz.

### Bi-quadratic filter

A bi-quadratic filter was applied to the system. A bi-quad filter has two poles and two zeros; it can be thought of as a high-pass filter in series with a low-pass filter. For these tests, the best case adjustment found experimentally was to set the high pass filter at 250 Hz and the low-pass filter at 100 Hz. The gains were raised again, this time to  $G_V = 8$  and  $G_{VTN} = 10$  mSec. Performance improved further. Settling time was reduced to 22 mSec. The bandwidth was measured as 56 Hz.

### Acceleration feedback

Acceleration feedback was applied [6 - 15]. Acceleration was observed using a Luenberger observer, as shown in Figure 8. The observer takes input from the motor current and the encoder. It adds the two and feeds the sum to a model of the motor. That model produces observed position, which is compared to actual position. The PID observer compensator drives out most error up to the observer bandwidth, which is usually between 200 and 500 Hz. One byproduct of the observer is an acceleration signal, which represents acceleration much better than double-differentiating the position feedback signal. A more complete discussion of the observer can be

found in [6, 9 - 11].

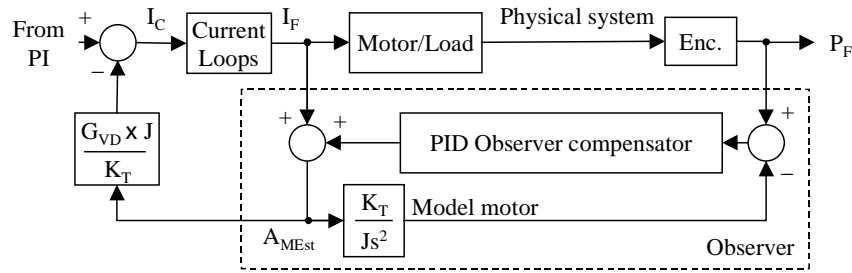


Figure 8. Luenberger observer provides observed acceleration ( $A_{MEst}$ ).

Acceleration feedback was applied to the bi-quad system. The gain  $G_{VD}$  was set to 15, which is approximately 2.5 in the SI units shown in Figure 8. This reduced the effects of resonance and the PI gains were raised ( $G_V = 22$  and  $G_{VTN} = 12$ ). The results were a dramatic improvement over the other systems. Settling time was reduced to just 12 mSec and the bandwidth was measured as 77 Hz.. The step response is shown in Figure 9.

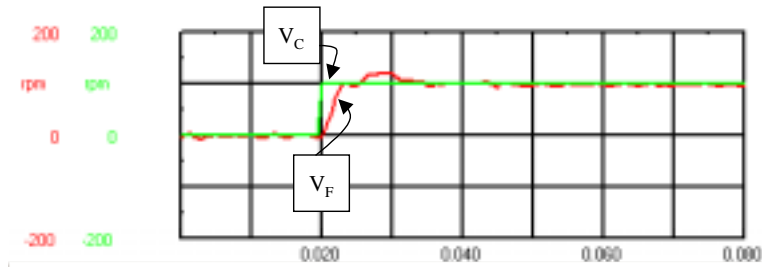


Figure 9. Step response of system with acceleration feedback.

## Comparison of all methods

All three methods (low-pass, bi-quad, and acceleration feedback/bi-quad) are compared in the frequency domain with the closed-loop Bode plot ( $V_F(s)/V_C(s)$ ) of Figure 10. This plot is generated by the ServoStar 600 Drive on the test mechanism of Figure 6. The results show the dramatic improvement offered by acceleration feedback. Notice that the bandwidth (the frequency where the gain falls to  $-3\text{dB}$ ) is greatly increased. In addition, stability margins are maintained. Peaking, the undesirable phenomenon where gain rises above 0dB at high frequency in the closed-loop response, is a reliable measure of stability. As shown in Figure 10, the peaking of all four configurations is about the same, with the base line system displaying the most peaking. This indicates that the cures for resonance allow higher gain while maintaining equivalent margins of stability.

## Conclusion

Low-frequency resonance is common in industry. It differs from the less common, but more often studied problem of high-frequency resonance. Low-frequency resonance causes oscillations at the first phase crossover the open-loop system; high frequency resonance causes oscillations at the natural mechanical frequency of the mechanism. Low-frequency resonance can be addressed by several methods including low-pass, bi-quadratic filters, and acceleration feedback. Acceleration feedback is a practical method for curing low-frequency resonance. It has been implemented in an industrial motor controller.

Acceleration feedback in combination with the bi-quad filter provides dramatic improvement for systems suffering from low-frequency resonance. This was demonstrated on laboratory hardware. Compared to the traditional solution of a single-pole low-pass filter, the combination of acceleration feedback and the bi-quad filter allow the settling time to be cut by a factor of three (from 35 mSec to 12 mSec) and the bandwidth to be raised by that same factor (23 Hz to 77 Hz). At the same time, acceleration feedback maintained stability margins, indicating that the increased gain of the servo system will be useful in practical applications.

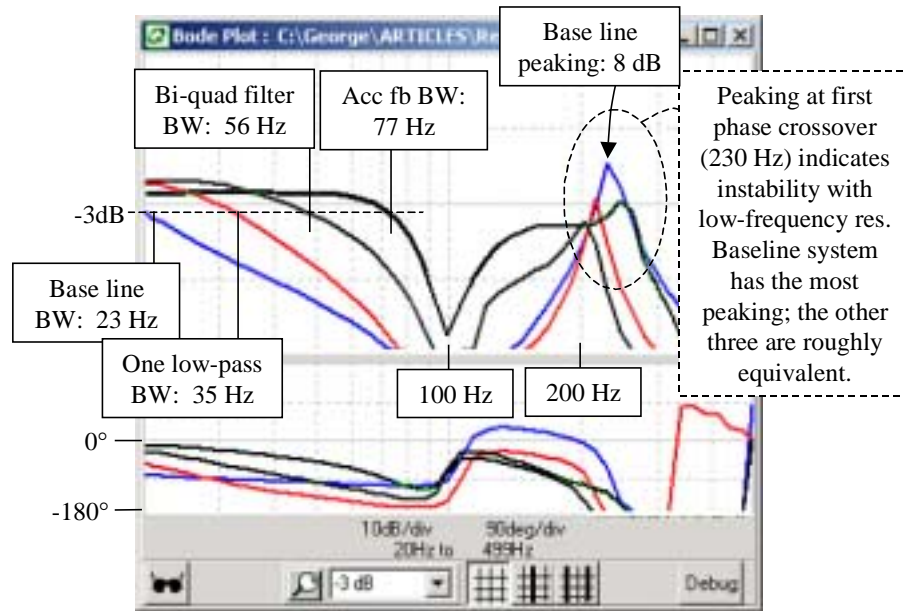


Figure 10. Comparison of closed-loop responses of baseline and when using three anti-resonance methods.

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